Original Research Article

MIXED SEMI-PRE FUZZY TOPOLOGICAL SPACES

Diganta Jyoti Sarma\textsuperscript{a} | Binod Chandra Tripathy\textsuperscript{b}

**ABSTRACT**

In this article we construct a fuzzy topology on a non-empty set $X$ called mixed semi-pre fuzzy topology from two given fuzzy topological spaces on $X$ with the help of fuzzy semi-pre quasi-neighborhood of a fuzzy point.

**Keywords:** fuzzy semi-preopen, fuzzy preopen, fuzzy semi-pre-q-nbd

**AUTHOR AFFILIATION**

\textsuperscript{a}Central Institute of Technology, Kokrajhar, BTAD, ASSAM-783370, INDIA
\textsuperscript{b}Department of Mathematics, Tripura University, Agartala - 799022, Tripura, INDIA

**CORRESPONDENCE**

\textsuperscript{a}Diganta Jyoti Sarma, Central Institute of Technology, Kokrajhar, BTAD, ASSAM-783370, INDIA
Email: dj.sarma@cit.ac.in
\textsuperscript{b}Binod Chandra Tripathy
Department of Mathematics, Tripura University, Agartala - 799022, Tripura, INDIA
Email: binodtripathy@tripurauniv.in

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1. INTRODUCTION

Zadeh [1] introduced the concept of fuzzy set. Based on this notion Chang [2] defined the notion of fuzzy topological spaces on a non-empty fuzzy set. Thereafter several mathematician turned their attention in the fuzzy setting of the theory of general topological spaces by using various types of nearly open sets of topological spaces. As a result, the notion of fuzzy semi-preopen sets has been introduced by Park and Lee [3] and Thakur et al. [4] and established some structural properties. They have also constructed the quasi neighborhood structure of a fuzzy point with the help of this type of sets. Using the notion of fuzzy semi-pre quasi neighborhood of a fuzzy point, Sarma and Tripathy [5] have established some additional properties and also defined this notion in product fuzzy topological spaces with some characterizations and have established some of their properties. Das and Baishya [6], defined the notion of mixed fuzzy topology. Thereafter a different type of mixed fuzzy topological space was introduced and its different properties have been investigated by Tripathy and Ray [7-10].

In this paper, we have introduced and studied the concept of mixed fuzzy topology with the help of fuzzy semi-preopen sets with some structural properties.

2. PRELIMINARIES

Throughout, we shall denote the fuzzy topological space on a set $X$ by $(X,\tau)$. Also, by $int(U)$, $cl(U)$ and $U$; we can denote respectively the interior, closure and complement of a fuzzy subset $U$ of a fuzzy topological space.

We recall some existing definitions and result those will be used in this sequel.

According to Zadeh [1], A fuzzy set $U$ in a non-empty set $X$ with a unit interval $I = [0,1]$ is a function $\lambda_U: X \rightarrow I$, where $\lambda_U$ is the membership function of $U$ and $\lambda_U(a)$ is the membership grade of $a$ in $U$. The union and intersection of two fuzzy sets $U$ and $V$ of $X$ can be defined by $(U\cup V)(a) = \max(U(a), V(a))$ and $(U\cap V)(a) = \min(U(a), V(a))$ respectively for all $a \in X$. For a family $\{U_i : i \in I\}$ of fuzzy sets in $X$, the union and intersection are defined by $\cup_{i \in I} U(a) = \sup\{U_i(a) : i \in I\}$ and $\cap_{i \in I} U(a) = \inf\{U_i(a) : i \in I\}$ respectively for all $a \in X$. Again if $U$ and $V$ are two fuzzy sets in $X$ and $Y$ respectively, then $U \times V$ is a fuzzy set in $X \times Y$ defined by $(U \times V)(a,b) = \min(U(a), V(b))$ for all $a \in U$ and for all $b \in V$.
According to Ming and Ming [11], a fuzzy point in X is a fuzzy set in X which is zero everywhere except at one point say a where it takes the value $\alpha \in (0,1]$ i.e. $0<\alpha<1$ and it is denoted by $a_\alpha$. If $U$ is any fuzzy set, then $a_\alpha \in U$ either means that the fuzzy point $a_\alpha$ takes its non-zero value in $(0,1)$ at the support $a$ and $\alpha < U(a)$ or fuzzy point $a_\alpha$ takes its non-zero value in $(0,1]$ and $\alpha \leq U(a)$. In particular, $a_\alpha \in U$ means $\alpha < U(a)$ and $0 < \alpha < 1$. A fuzzy point $a_\alpha$ is said to be quasi-coincident (in short, $q$-coincident) with a fuzzy set $U$ in $X$ denoted by $a_\alpha U$ if $\alpha + U(a) > 1$ or $\alpha > |U(a)|$.

A fuzzy set $U$ is said to be quasi-coincident with another fuzzy set $V$ denoted by $UqV$ if and only if there exists an $a \in X$ such that $U(a) + V(a) > 1$. If a fuzzy point $a_\alpha$ is not quasi-coincident with a fuzzy set $U$, then we can denote it by $a_\alpha qU$. A fuzzy set $U$ is said to be quasi-neighborhood (in short, $q$-nbd) of a fuzzy point $a_\alpha$ if there exists a fuzzy open set $V$ in $X$ such that $a_\alpha qV \subseteq U$.

**Definition 2.1.** Let $(X, \tau)$ be a fuzzy topological space. Then a fuzzy subset $U$ in $X$ is said to be

(a) [12] fuzzy preopen (respectively, fuzzy preclosed) if $U \subseteq int(clU)$ (respectively, $U \supseteq cl(intU)$).

(b) [3, 4] fuzzy semi-preopen (respectively, fuzzy semi-preclosed) if there exists a fuzzy preopen set (respectively, fuzzy preclosed set) $V$ in $X$ such that $U \subseteq V \subseteq clU$ (respectively, $intU \subseteq U \subseteq V \subseteq clU$).

It is obvious that every fuzzy open set is fuzzy preopen and every fuzzy preopen set is fuzzy semi-preopen. Semicontinuous functions in fuzzy setting has been studied by Azad [13] and fuzzy preseparation axioms by Singal and N. Prakash [14].

**Definition 2.2. [3]** The semi-preclosure and semi-preinterior of a fuzzy set $U$ in a fuzzy topological space $(X, \tau)$ is denoted by $spcl(U)$ and $spint(U)$ respectively and are given by

$$
spcl(U) = \{ V : V \in \mathcal{U} \text{ and } V \text{ is fuzzy semi-preclosed in } X \},
$$

$$
spint(U) = \{ V : V \in \mathcal{U} \text{ and } V \text{ is fuzzy semi-preopen in } X \}.
$$

**Definition 2.3. [3]** A fuzzy set $U$ in a fuzzy topological space $(X, \tau)$ is said to be a fuzzy semi-pre quasi neighborhood (in short, fuzzy semi-pre $q$-nbd) of a fuzzy point $a_\alpha$ if there exists a fuzzy semi-preopen set $V$ in $X$ such that $a_\alpha qV \subseteq U$.

The following results of Sarma and Tripathy [5] will be used for establishing some results of this article.

**Lemma 2.1. (Sarma and Tripathy [5]).** If $U$ and $V$ be two fuzzy semi-pre-$q$-nbd of a fuzzy point $a_\alpha$ in a fuzzy topological space $(X, \tau)$, then $U \land V$ is also a fuzzy semi-pre-$q$-nbd of $a_\alpha$ satisfying the condition that intersection of two fuzzy semi-preopen sets is fuzzy semi-preopen.

**Lemma 2.2. (Sarma & Tripathy [5]).** Let $(X, \tau_1)$ and $(X, \tau_2)$ be two fuzzy topological spaces. If every fuzzy $\tau_1$-semi-pre-$q$-nbd of a fuzzy point $a_\alpha$ is $\tau_2$-semi-pre-$q$-nbd of $a_\alpha$ then $\tau_1 \leq \tau_2$.

**Definition 2.4. (Sarma and Tripathy [5]).** Let $X$ and $Y$ be two fuzzy topological spaces such that $X$ is product related to $Y$. A fuzzy point in $X \times Y$ is a fuzzy set in $X \times Y$, which is $(0,0) \in X \times Y$ everywhere except at one point say $(a,b) \in X \times Y$ where it takes value say $\alpha = min(y,\delta)$, $(0<\gamma \leq 1, 0<\delta \leq 1)$ and it is denoted by $(a,b)_\alpha$, with its support $(a,b)$.

**Definition 2.5. (Sarma and Tripathy [5]).** Let $X$ and $Y$ be two fuzzy topological spaces such that $X$ is product related to $Y$. Then a fuzzy set $U_x \times V_y$ in $X \times Y$ is said to be a fuzzy semi-pre-$q$-nbd of a fuzzy point $(a,b)_\alpha$ if there exists a fuzzy semi-preopen set $V_x \times V_y$ in $X \times Y$ such that $(a,b)_\alpha q(V_x \times V_y)$ and $V_x \times V_y \subseteq U_x \times U_y$.

### 3. Mixed Semi-pre Fuzzy Topological Space

In this section, we have constructed the mixed fuzzy topology with the help of fuzzy semi-pre quasi neighborhood of a fuzzy point.

**Definition 3.1.** Let $(X, \tau_1)$ and $(X, \tau_2)$ be two fuzzy topological spaces on $X$. Define $\tau_1(\tau_2) = \{ U \in \mathcal{P}^X : \text{for every fuzzy point } a_\alpha qU, \text{ there exists a fuzzy } \tau_2\text{-semi-pre-}q\text{-}nbd } U_a \text{ of } a_\alpha \text{ such that } \tau_1 \text{-spcl}(U_a) \subseteq U \}$ Then $\tau_1(\tau_2)$ is a fuzzy topology on $X$ under the condition that intersection of two fuzzy semi-preopen sets is fuzzy semi-preopen. This topology is called mixed semi-pre fuzzy topology.

**Theorem 3.1.** If $\tau_1(\tau_2) = \{ U \in \mathcal{P}^X : \text{for every fuzzy point } a_\alpha qU, \text{ there exists a fuzzy } \tau_2\text{-semi-pre-}q\text{-}nbd } U_a \text{ of } a_\alpha \text{ such that } \tau_1 \text{-spcl}(U_a) \subseteq U \}$ Then $\tau_1(\tau_2)$ is a fuzzy topology on $X$ under the condition that intersection of two fuzzy semi-preopen sets is fuzzy semi-preopen.
Proof. (i) Here 1ₓ ∈ τ₁(τ₂). Since α + 1ₓ > 1, so for any fuzzy point aₓ aₓq₁ₓ and for any fuzzy τ₂-semi-pre-q-nbd Uₙ = Iₓ of aₓ τ₁-spcl(Uₙ) = τ₁-spcl(Iₓ) = Iₓ. Also, 0ₓ ∈ τ₁(τ₂), since 0ₓ is not q-coincident with any fuzzy point for α + 0ₓ > 1 and so there does not exist any question for violating the condition of being a member of τ₁(τ₂).

Hence 1ₓ 0ₓ ∈ τ₁(τ₂).

(ii) Let {Uₜ : λ ∈ λ} be a collection of fuzzy semi-preopen sets in τ₁(τ₂).

To show that Vₖₑₐ Uₜ ∈ τ₁(τ₂).

Suppose that for any fuzzy point aₓ aₓqₖₑₐ Uₜ. This implies α + Vₖₑₐ Uₜ(a) > 1. Then we have α + Sup Vₖₑₐ Uₜ(a) > 1. Therefore there exists a λₒ ∈ λ such that α + Uₜλₒ(a) > 1. So, aₓqₜλₒ. Since, Uₜλₒ ∈ τ₁(τ₂), so there exists a fuzzy τ₂-semi-pre-q-nbd Vₜλₒ of aₓ such that τ₁-spcl(Vₜλₒ) ≤ Uₜλₒ ≤ Vₖₑₐ Uₜ.

Thus for each fuzzy point aₓqₖₑₐ Uₜ, there exists a fuzzy τ₂-semi-pre-q-nbd Vₜλₒ of aₓ such that τ₁-spcl(Vₜλₒ) ≤ Vₖₑₐ Uₜ.

Hence Vₖₑₐ Uₜ ∈ τ₁(τ₂).

(iii) Let U₁U₂ ∈ τ₁(τ₂). To show that U₁ ∧ U₂ ∈ τ₁(τ₂).

Suppose that aₓq(U₁ ∧ U₂). So α + (U₁ ∧ U₂)(a) > 1.
⇒ α + min(U₁(a), U₂(a)) > 1
⇒ α + U₁(a) > 1
and
α + U₂(a) > 1
⇒ aₓqU₁ and aₓqU₂.

Since U₁ ∈ τ₁(τ₂), so for any fuzzy point aₓqU₁, there exists a fuzzy τ₂-semi-pre-q-nbd V₁ of aₓ such that τ₁-spcl(V₁) ≤ U₁. Also U₂ ∈ τ₁(τ₂), so for any fuzzy point aₓqU₂, there exists a fuzzy τ₂-semi-pre-q-nbd V₂ of aₓ such that τ₁-spcl(V₂) ≤ U₂. Since V₁ and V₂ are fuzzy τ₂-semi-pre-q-nbd of aₓ so by Lemma 2.1 we have V₁ ∧ V₂ is also a fuzzy τ₂-semi-pre-q-nbd of aₓ.

Also (τ₁-spcl(V₁)) ∧ (τ₁-spcl(V₂)) ≤ U₁ ∧ U₂.

Let aₓ ∈ τ₁-spcl(V₁ ∧ V₂). Then by Theorem 2.9 of [6], every fuzzy τ₁-semi-pre-q-nbd V of aₓ is q-coincident with V₁ ∧ V₂.

That is V q(V₁ ∧ V₂).
⇒ V(a) + (V₁ ∧ V₂)(a) > 1.
⇒ V(a) + min(V₁(a), V₂(a)) > 1.
⇒ V(a) + V₁(a) > 1 and V(a) + V₂(a) > 1.
⇒ V qV₁ and V qV₂.
⇒ aₓ ∈ τ₁-spcl(V₁) and aₓ ∈ τ₁-spcl(V₂).
⇒ aₓ ∈ τ₁-spcl(V₁) ∧ τ₁-spcl(V₂).

Consequently, τ₁-spcl(V₁ ∧ V₂) ≤ τ₁-spcl(V₁) ∧ τ₁-spcl(V₂) ≤ U₁ ∧ U₂.

Thus for every fuzzy point aₓq(U₁U₂) there exists a fuzzy τ₂-semi-pre-q-nbd V₁V₂ of aₓ such that τ₁-spcl(V₁ ∧ V₂) ≤ U₁ ∧ U₂.

Therefore U₁ ∧ U₂ ∈ τ₁(τ₂). Hence τ₁(τ₂) is fuzzy topology on X.

Now we define fuzzy semi-pre quasi neighborhood of a fuzzy point in mixed semi-pre fuzzy topological space (X, τ₁(τ₂)) as following.

Definition 3.2. A fuzzy set U in a mixed semi-pre fuzzy topological space (X, τ₁(τ₂)) is said to be a fuzzy τ₁(τ₂)-semi-pre quasi-
neighborhood (in short, fuzzy τ₁(τ₂)-semi-pre-q-nbd) of a fuzzy point aₓ if there exists a fuzzy τ₂-semi-preopen set V in X such that aₓqV and τ₁-spcl(V) ≤ U.

Theorem 3.2. Let (X, τ₁(τ₂)) be a mixed semi-pre fuzzy topological space. If Uₙₑₐ = {U ∈ Pₖ : for every fuzzy point aₓ there exists a fuzzy τ₂-semi-preopen set V in X such that aₓqV and τ₁-spcl(V) ≤ U}, then there exists a fuzzy topology τ₁(τ₂) with respect
to which \( U_{ao} \) is a fuzzy semi-pre quasi-neighborhood system of \( a_o \) under the condition that intersection of two fuzzy semi-preopen sets is fuzzy semi-preopen.

**Proof.** (i) Let \( a_o \) be a fuzzy point and \( U \in U_{ao} \). To show that \( a_o q U \).

Since \( U \in U_{ao} \) so there exists a fuzzy \( \tau_2 \)-semi-preopen set \( V \) in \( X \) such that \( a_o q V \) and \( \tau_1 \)-spcl\( (V) \leq U \). Now \( a_o q V \) implies \( \alpha ^+ V (a) > 1 \).

Also, \( V \subseteq \tau_1 \)-spcl\( (V) \) \( \leq U \). So \( V(a) \leq U(a) \). Now \( \alpha ^+ U(a) \geq \alpha ^+ V (a) > 1 \) which implies \( \alpha ^+ U(a) > 1 \) and so we have \( a_o q U \).

(ii) Let \( U, V \in U_{ao} \). Then there exists fuzzy \( \tau_2 \)-semi-preopen sets \( U_1 \) and \( V_1 \) such that \( a_o q U_1 \) with \( \tau_1 \)-spcl\( (U_1) \) \( \leq U \) and \( a_o q V_1 \) with \( \tau_1 \)-spcl\( (V_1) \) \( \leq V \) respectively.

Now \( a_o q U_1 \) and \( a_o q V_1 \).

\[ \Rightarrow \alpha ^+ U_1(a) > 1 \text{ and } \alpha ^+ V_1(a) > 1 \]
\[ \Rightarrow \alpha ^+ \min(U_1(a), V_1(a)) > 1 \Rightarrow \alpha ^+ (U_1 \land V_1)(a) > 1 \]
\[ \Rightarrow a_o q (U_1 \land V_1) \]

Since \( U_1 \) and \( V_1 \) are fuzzy \( \tau_2 \)-semi-preopen sets, so by the given condition \( (U_1 \land V_1) \) is fuzzy \( \tau_2 \)-semi-preopen.

Also, \( \tau_1 \)-spcl\( (U_1 \land V_1) \) \( \subseteq \tau_1 \)-spcl\( (U_1) \land \tau_1 \)-spcl\( (V_1) \) \( \leq U \land V \). Hence \( U \land V \in U_{ao} \).

(iii) Let \( U \) and \( V \) be two fuzzy sets such that \( U \in U_{ao} \) and \( U \leq V \). To show that \( V \in U_o \).

Since \( U \in U_{ao} \) so there exists a fuzzy \( \tau_2 \)-semi-preopen set \( U_1 \) such that \( a_o q U_1 \) and \( \tau_1 \)-spcl\( (U_1) \) \( \leq U \). Hence \( V \in U_{ao} \).

Thus \( \tau_1 (\tau_2) \) is a fuzzy topology on \( X \) for which \( U_{ao} \) is a fuzzy semi-pre Quasi neighborhood system of \( a_o \).

**Theorem 3.3.** Let \((X, \tau_1(\tau_2))\) be a mixed semi-pre fuzzy topological space where \((X, \tau_1)\) and \((X, \tau_2)\) are two fuzzy topological spaces. Then \( \tau_1 (\tau_2) \subseteq \tau_2 \).

**Proof.** Let \( U \) be a fuzzy \( \tau_1 (\tau_2) \)-semi-pre-q-nbd of a fuzzy point \( a_o \). Then there exists a fuzzy \( \tau_2 \)-semi pre-open set \( W \) in \( X \) such that \( a_o q W \) and \( \tau_1 \)-spcl\( (W) \leq U \). Now \( W \) is fuzzy \( \tau_2 \)-semi-preopen set such that \( a_o q W \) and \( \tau_2 \)-spcl\( (W) \leq U \). Since \( U \) is also a fuzzy set, so by definition \( U \) is fuzzy \( \tau_2 \)-semi-pre-q-nbd of \( a_o \). Hence by Lemma 2.2, we have \( \tau_1 (\tau_2) \subseteq \tau_2 \).

**Definition 3.3.** A fuzzy topological space \((X, \tau)\) is said to be fuzzy semi-pre regular if for each fuzzy point \( a_o \) and each fuzzy semi-preclosed set \( W \) with \( a_o q (1 - W) \), there exists a fuzzy open set \( A \) and a fuzzy semi-preopen set \( B \) such that \( a_o q A, W \leq B \) and \( A \) is not q-coincident with \( B \).

**Theorem 3.4.** Let \((X, \tau_1(\tau_2))\) be a mixed semi-pre fuzzy topological space where \((X, \tau_1)\) and \((X, \tau_2)\) are two fuzzy topological spaces such that \((X, \tau_1)\) is fuzzy semi-pre regular and \( \tau_1 \subseteq \tau_2 \). Then \( \tau_1 \subseteq \tau_1 (\tau_2) \).

**Proof.** Let \( U \) be a fuzzy \( \tau_1 \)-semi-pre-q-nbd of a fuzzy point \( a_o \). Then there exists a fuzzy \( \tau_1 \)-semi-preopen set \( A \) in \( X \) such that \( a_o q A \) and \( A \leq U \).

Now, for any fuzzy \( \tau_1 \)-semi-preopen set \( A, a_o q A \) implies that \( a_o q (1 - A^c) \) where \( A^c \) is fuzzy \( \tau_1 \)-semi-preclosed.

Since \((X, \tau_1)\) is fuzzy semi-pre regular, so for every fuzzy point \( a_o \), there exists a fuzzy \( \tau_1 \)-open set \( B \) and a fuzzy \( \tau_1 \)-semi-preopen set \( V \) such that \( a_o q B, A^c \leq V \) and \( B q V \).

We know that \( B \) is fuzzy \( \tau_1 \)-open implies \( B \) is fuzzy \( \tau_1 \)-preopen which again implies \( B \) is fuzzy \( \tau_1 \)-semi-preopen. Then \( \tau_1 \)-spcl\( (B^c) \) \( q V \). Hence \( a_o q B \leq \tau_1 \)-spcl\( (B^c) \) \( \leq V^c \leq A \).

Now, \( \tau_1 \)-spcl\( (B) \leq A \leq U \) implies that \( \tau_1 \)-spcl\( (B) \leq U \).

Thus for every fuzzy point \( a_o \), there exists a fuzzy \( \tau_2 \)-semi-preopen set \( B \) quasi-coincident with \( a_o \) such that \( \tau_1 \)-spcl\( (B) \leq U \).

Therefore \( U \) is fuzzy \( \tau_1 (\tau_2) \)-semi-pre-q-nbd of \( a_o \). Hence by Lemma 2.2, we have \( \tau_1 \subseteq \tau_1 (\tau_2) \).
Lemma 3.1. (Sarma and Tripathy [7]) Let X and Y be two fuzzy topological spaces such that X is product related to Y. If U and V be two fuzzy semi-pre-q-nbd of a₁ in X and b₁ in Y respectively, then U× V is also a fuzzy semi-pre-q-nbd of (a₁, b₁) in X× Y, where α = min(γ, δ).

Lemma 3.2. (Sarma and Tripathy [7]) Let a₁ and b₁ are two fuzzy points in fuzzy topological space X and Y respectively and A× B be a fuzzy set in X× Y. Then (a₁, b₁) ∈ spcl(A× B) if and only if for every fuzzy semi-pre-q-nbd U× V of (a₁, b₁), (U× V)q(A× B), where α = min(γ, δ).

Theorem 3.5. Let (X, τ₁(τ₁)) and (Y, τ₃(τ₃)) be two mixed semi-pre fuzzy topological spaces where (X, τ₁), (X, τ₂), (Y, τ₃), and (Y, τ₄) be fuzzy topological spaces. Then τ₁(τ₂)× τ₃(τ₄) ≤ τ₁(τ₁)(τ₂× τ₄).

Proof. Let (U× V) ≤ τ₁(τ₂)× τ₃(τ₄). Then U ∈ τ₁(τ₂) and V ∈ τ₃(τ₄).
Suppose that (a₁, b₁) ∈ q(U× V), where α = min(γ, δ). Then we have α + (U× V)(a₁, b₁) > 1.
⇒ min(γ, δ) + min(U(a₁), V(b₁)) > 1
⇒ γ + U(a₁) > 1 and δ + V(b₁) > 1.
⇒ a₁q U and hence b₁q V.
Since, U ∈ τ₁(τ₂), so for every fuzzy point a₁q U, there exists a fuzzy τ₂-semi-pre-q-nbd Uᵣ of a₁ such that τ₁-spcl(Uᵣ) ≤ U.
Also, V ∈ τ₃(τ₄), so for every fuzzy point b₁q V, there exists a fuzzy τ₄-semi-pre-q-nbd Vᵣ of b₁ such that τ₃-spcl(Vᵣ) ≤ V.
Therefore by Lemma 3.1, we have Uᵣ× Vᵣ is a fuzzy (τ₂× τ₄)-semi-pre-q-nbd of (a₁, b₁), such that τ₁-spcl(Uᵣ)× τ₃-spcl(Vᵣ) ≤ U× V.
Now we show that (τ₁× τ₃)-spcl(Uᵣ× Vᵣ) ≤ τ₁-spcl(Uᵣ)× τ₃-spcl(Vᵣ).
Let (a₁, b₁) ∈ (τ₁× τ₃)-spcl(Uᵣ× Vᵣ). Also let Uᵣ₁ and Vᵣ₃ are fuzzy τ₁-semi-pre-q-nbd of a₁ and fuzzy τ₃-semi-pre-q-nbd of b₁ respectively and α₁ = min(γ₁, δ₁). Therefore Uᵣ₁× Vᵣ₃ is fuzzy (τ₁× τ₃)-semi-pre-q-nbd of (a₁, b₁).
Since, Uᵣ and Vᵣ are fuzzy sets in X and Y respectively, so Uᵣ× Vᵣ is a fuzzy set in X× Y.

We have, (a₁, b₁) ∈ (τ₁× τ₃)-spcl(Uᵣ× Vᵣ), therefore by Lemma 3.2, we have for every fuzzy (τ₁× τ₃)-semi-pre-q-nbd Uᵣ₁× Vᵣ₃ of (a₁, b₁), (Uᵣ₁× Vᵣ₃)q(Uᵣ× Vᵣ).
This implies, min{Uᵣ₁(a₁), Vᵣ₃(b₁)} + min{Uᵣ₁(a₁), Vᵣ₃(b₁)} > 1
⇒ Uᵣ₁(a₁) + Uᵣ₁(a₁) > 1 and Vᵣ₃(b₁) + Vᵣ₃(b₁) > 1
⇒ Uᵣ₁q Uᵣ and Vᵣ₃q Vᵣ₃.
Next, by Theorem 2.9 of [6], a₁ ∈ τ₁-spcl(Uᵣ) and b₁ ∈ τ₃-spcl(Vᵣ).
⇒ a₁× b₁ ∈ τ₁-spcl(Uᵣ)× τ₃-spcl(Vᵣ)
⇒ (a₁, b₁) ∈ τ₁-spcl(Uᵣ)× τ₃-spcl(Vᵣ).
Therefore, (τ₁× τ₃)-spcl(Uᵣ× Vᵣ) ≤ τ₁-spcl(Uᵣ)× τ₃-spcl(Vᵣ) ≤ U× V.

Thus for every fuzzy point (a₁, b₁) ∈ q(U× V), there exists a (τ₁× τ₃)-semi-pre-q-nbd Uᵣ× Vᵣ of (a₁, b₁) such that (τ₁× τ₃)-spcl(Uᵣ× Vᵣ) ≤ U× V.

Consequently, U× V ∈ (τ₁× τ₃)(τ₂× τ₄).
Hence, τ₁(τ₂)× τ₃(τ₄) ≤ (τ₁× τ₃)(τ₂× τ₄).
CONFLICT OF INTERESTS

The authors declare that there is no conflict of interest related to the publication of this article.

REFERENCES


